Math 116 Section 04 ***SOLUTIONS***

Midterm 1 Name ____________________

August 03, 2005

Instructor: Charles Cuell Student Number ______________

All solutions are to be presented on the paper in the space provided. The exam is closed book, no calculators. Time for the exam is 50 minutes.

(1) (**5 marks each**) Evaluate the following:

(a) $\displaystyle\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

(b) $\displaystyle\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$

(2) (**5 marks**) Write the definition of the definite integral of $f(x)$ over the interval $x_0 \leq x \leq x_1$.

$$\int_{x_0}^{x_1} f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x$$

(3) (**5 marks**) Let $f(x) = \displaystyle\int_{-1}^{x^2} \sqrt{1+t^2}\,dt$. What is $f'(x)$?

$$f'(x) = \sqrt{1+x^4}\;2x$$

(4) (**2 marks each**) Evaluate the follwing indefinite integrals:

(a) $\displaystyle\int e^x\,dx = e^x + C$

(b) $\displaystyle\int a^x\,dx = \frac{a^x}{\ln a} + C$

(c) $\displaystyle\int \sec^2 x\,dx = \tan x + C$

(d) $\displaystyle\int \csc x \cot x\,dx = -\csc x + C$

(e) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1} x + C$

(5) (**5 marks each**) Evaluate the following definite integrals:

(a) $\int_1^{16} x^{\frac{1}{4}}\,dx$

$$\begin{aligned}\int_1^{16} x^{\frac{1}{4}}\,dx &= \frac{4}{5}x^{\frac{5}{4}}\Big|_1^{16} \\ &= \frac{4}{5}(16^{\frac{5}{4}} - 1) \\ &= \frac{4}{5}31\end{aligned}$$

(b) $\int_1^2 \frac{x^2+\sqrt{x}}{x^3}\,dx$

$$\begin{aligned}\int_1^2 \frac{x^2+\sqrt{x}}{x^3}\,dx &= \int_1^2 \left(\frac{1}{x} + x^{\frac{-5}{2}}\right)\,dx \\ &= \left(\ln x - \frac{2}{3}x^{\frac{-3}{2}}\right)\Big|_1^2 \\ &= \left(\ln 2 - \frac{2}{3}2^{\frac{-3}{2}}\right) - \left(\ln 1 - \frac{2}{3}1^{\frac{-3}{2}}\right) \\ &= \left(\ln 2 - \frac{2}{3}2^{\frac{-3}{2}}\right) + \frac{2}{3}\end{aligned}$$

(c) $\int_{-1}^1 |x|\,dx$

$$\begin{aligned}\int_{-1}^1 |x|\,dx &= \int_{-1}^0 (-x)\,dx + \int_0^1 x\,dx \\ &= -\frac{x^2}{2}\Big|_{-1}^0 + \frac{x^2}{2}\Big|_0^1 \\ &= -\left(0 - \frac{(-1)^2}{2}\right) + \left(\frac{1^2}{2} - 0\right) \\ &= \frac{1}{2} + \frac{1}{2} \\ &= 1\end{aligned}$$

(6) (**5 marks**) Evaluate the indefinite integral: $\displaystyle\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx$

Let $u = \sin^{-1} x$. Then $du = \dfrac{1}{\sqrt{1-x^2}} dx$ and

$$\begin{aligned}
\int \frac{\sin^{-1} x}{\sqrt{1-x^2}}\,dx &= \int u\,du \\
&= \frac{u^2}{2} + C \\
&= \frac{(\sin^{-1} x)^2}{2} + C
\end{aligned}$$

(7) (**5 marks**) Evaluate the indefinite integral: $\displaystyle\int_0^{\sqrt{\pi}} x \sin x^2\,dx$

Let $u = x^2$. Then $du = 2x dx$, $u(0) = 0$, $u(\sqrt{\pi}) = \pi$ and

$$\begin{aligned}
\int_0^{\sqrt{\pi}} x \sin x^2\,dx &= \frac{1}{2}\int_0^{\pi} \sin u du \\
&= -\frac{1}{2} \cos u\Big|_0^{\pi} \\
&= -\frac{1}{2} (\cos(\pi) - \cos(0)) \\
&= -\frac{1}{2}(-1-1) \\
&= 1
\end{aligned}$$